# Evaluation of Estonian Research

# - Mathematical Sciences -

Report to the Estonian Science Fund Council

by

Professor Torsten Ekedahl,
Department of Mathematics, Stockholm University
Professor Lars Inge Hedberg
Department of Mathematics, Linköping University
Professor Lars Holst
Mathematical Statistics,
Royal Institute of Technology, Stockholm
and
Professor Axel Ruhe
Computer Science, Chalmers University of Technology, Göteborg

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# Foreword

Several Swedish organizations have been asked to take part in a general evaluation of all research performed at academic institutions in Estonia. NFR has agreed to organize the evaluation of Estonian research within the field of natural science. This report has been prepared according to an agreement between the Estonian Science Fund Council and the Swedish Natural Science Research Council (NFR).

During the spring of 1991 Estonian scientists completed reports on their research which were sent to NFR. These reports have subsequently been distributed among 14 Swedish evaluation groups. In total about 40 Swedish scientists are engaged in the evaluations. The groups are making site visits to the Estonian laboratories and institutes during 1991/92 to discuss the research performed, the plans for future activities and to get information about the working conditions, experimental facilities, financial resources etc. Each group has been instructed to produce a report assessing its particular research area.

This report concerns the sub-field of mathematical sciences and will eventually be a part of an extensive report covering all Estonian research in natural science.

The organization of the site visits is done in close cooperation with the Estonian Science Fund Council. Although difficult times prevail in Estonia all the site visits have been successful. The NFR is grateful to the Estonian Science Fund Council for its efforts to handle all practical matters in connection with these visits.

The NFR is also grateful to the Swedish scientists who with enthusiasm and great skill have taken part in the demanding evaluation work.

Finally, the Council wishes to express its sincere hope that this evaluation report will contribute to a further positive development and strengthening of Estonian science.

Carl Nordling Secretary General

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# INTRODUCTION

The Estonian Science Fund Council has instructed Estonian scientists in the field of Mathematical Sciences to prepare reports concerning their research activities during the last 5 years. These reports were completed during the spring of 1991 and dealt with the following points:

- project leader(s)
- short description of the objectives
- summary of results
- summary of resources
- scientific staff and their qualifications
- list of publications
- dissertations
- scientific meetings organized
- prognosis of the future development of the project

In most cases the reports were accompanied by reprints of scientific publications written in English and Russian.

In July 1991 the reports were sent to NFR and during the autumn the evaluators received the reports. Site visits by the evaluators to the research groups in Tallinn and Tartu were done in the period March 16-19 1992. The Estonian Science Fund Council had appointed Rein Lepik as organizer and contact person for this evaluation.

# **ACKNOWLEDGEMENTS**

We want to express our great appreciation to our Estonian colleagues for their carefully prepared written reports, and for the many interesting meetings and discussions we had during our short visit to Estonia. We are especially grateful to Rein Lepik, who had been able to organize the visit in an excellent way in spite of the difficult times. We hope that our mission will contribute to the creation of future close contacts between mathematicians in our countries.

# GENERAL COMMENTS ON THE MATHEMATICAL SCIENCES

The group's assignment was to evaluate Estonian research in the mathematical sciences. To be evaluated were projects from pure and applied mathematics, numerical analysis, statistics, and theoretical mechanics. Unfortunately no project in computer science proper was included in the evaluation. On the other hand, the inclusion of theoretical mechanics among the subjects caused us some problems, as the group did not include any specialist in that area.

Generally speaking our impressions of Estonian mathematics are rather mixed. We found several examples of mathematical research on a high international level, and of mathematicians with a well developed network of international contacts. On the other hand, we have the impression that Estonian mathematics suffers from a certain amount of provinciality. At least to an outsider this seems surprising in view of the fact that Estonia is geographically close to two of the major mathematical centres of the world, S:t Petersburg and Moscow, but the phenomenon seems to be common to many parts of the former Soviet Union.

It is certainly impossible for a country the size of Estonia to follow the international trends in all major branches of mathematics, but we were struck by the number of mathematicians who work in fields that are considered obsolete in most parts of the world, and where the international level of activity is very low. Some of this work can still be quite good, but for the sound future development of mathematics in Estonia a greater openness to new ideas is necessary.

Related to this is the problem of publication and the low international visibility of Estonian mathematics. A very large part of Estonian mathematical research has been published solely in Estonian publications or in Soviet publications with very limited circulation. We believe that we understand the reasons for this, and we have noticed that now many articles are published in English. However, we still want to emphasize the importance of publishing articles in good international journals. Through their refereeing system this will serve as a quality control, in addition to being a further means for increasing international contacts. It is perhaps also worth pointing out that the Estonian scientific journals, by not being specialized to mathematics, are usually not available in mathematical libraries, but only in less accessible central university libraries. A remedy for this would be to found a mathematical journal, but this would need a larger base than just Estonia.

In order to ensure a rapid development, and to overcome the inbreeding in Estonian mathematics, it is of decisive importance that the brightest young Estonian researchers should have the opportunity to spend a year or two at some major mathematical center abroad early in their mathematical career, for example as graduate students or as postdoctoral fellows. We are concerned that the construction of the new doctor's degree will work against this, since the expected time spent in attaining it will be so long that the Ph.D:s will be too old to be willing to change their direction of research. Generally, we feel that the total length of undergraduate and graduate programs should not be longer than eight years. (Parenthetically, we would like to add that one of the main complaints usually voiced by international evaluators of Swedish research is that Swedish Ph.D:s are too old.)

Another way of increasing international contacts is, of course, to bring foreign scientists to Estonia. The plans for the Tartu Mathematical Center, a small mathematical research institute in Tartu, where Estonian and foreign mathematicians may spend a limited time, seems to us to be a step in the right direction.

# **EVALUATION OF RESEARCH GROUPS**

Gennadi Vainikko Chair of Numerical Analysis Tartu University

Numerical Methods for Ordinary and Partial Differential Equations

# Principal activities

In this group several application motivated problems are studied, as well as numerical methods.

P Oja uses splines to solve boundary value problems for ordinary differential equations. The order of the error term is found for several variants of collocation and subregion splines. Numerical tests have been done to verify error expansions. E Tamme uses splines for parabolic initial boundary value problems. M Fischer studies implicit schemes for nonlinear parabolic problems. Several students have done diploma works on problems in these areas.

I-I Saarniit reported work on two important applied problems leading to parabolic equations, one arising from laser discharge, the other one from heat generation and conduction in a tyristor, in both cases in close collaboration with users in physics and the electrical industry.

O Karma deals with nonlinear eigenvalue problems in Banach spaces and studies convergence of the solutions to finite dimensional approximations to those of the continuous problem. P Miidla studies a special case of this, occurring when one seeks periodic solutions to nonlinear autonomous differential systems. Error expansions but no numerical tests were reported.

# Evaluation

This group applies well chosen numerical methods in a competent way. A strong point is the close connection to practical application, a weakness is the absence of numerical experiments in some cases. The overall rating is good.

# Recommendations

We strongly recommend support for this group. It is a way of building up competence in several practically important areas of numerical analysis in Estonia.

Gennadi Vainikko Chair of Numerical Analysis and Laboratory of Applied Mathematics Tartu University

Ill-posed Problems

# Principal activities

This is the main track of the works of G Vainikko, and six of his students have completed dissertations during the last five years. A unified framework which covers different schemes of finding a bounded approximation to the possibly unbounded inverse of a linear operator between two Hilbert spaces is developed. It covers Tikhonov regularization as well as various stationary iteration schemes.

T Raus has studied how solutions obtained by applying a discrepancy principle when choosing a regularization parameter (minimizing a weighted average of the residual and the solution norm) can be regarded as quasi optimal.

U Hämarik studies the problem for which choices of finite dimensional subspaces the orthogonal projection becomes regularized. This is of interest when doing discretization prior to numerical solution.

G Vainikko also discussed how theories developed in this project can be put to use for finding solutions to the non smooth case of the ground water filtration problem. This will be done in a large Estonian collaboration.

# Evaluation

The work in this group is of very good quality and well known internationally. It has important applications.

# Recommendations

Support is strongly recommended. Numerical experiments on well chosen problems may give insights that lead to further developments.

Gennadi Vainikko Chair of Numerical Analysis and Laboratory of Applied Mathematics Tartu University

Weakly Singular Integral Equations

# Principal activities

This group, led by G Vainikko, has worked for a long time and studied several aspects on the numerical solution of integral equations. Interest is centered on the case when the kernel has a singularity at the diagonal. This shows up in the solution in several places, and people in this group have obtained a keen feeling for how this comes about.

G Vainikko reported on smoothness results for both the solution and its normal and tangential derivatives in the two dimensional case. U. Kangro reported on the case with a non smooth boundary, where she has shown that singularities in the shape of the boundary curve show up as singularities in the solution only at those singular points. A Pedas reported a comparison between several numerical techniques, including spline collocation and interpolatory quadrature, comparing accuracy estimates as well as performance on a practical numerical experiment.

P Uba has studied a one dimensional problem where the singularity is along a curve instead of at the diagonal, and developed a non uniform grid collocation method for that case. R Kangro has investigated where the solution gets singular in those generalized cases.

# Evaluation

The work is of a very good quality. A whole list of interesting directions for further work is given, and the group certainly has the expertise necessary to reach interesting results along those lines.

The evaluation group was impressed by the breadth and depth of the works presented in all the three research groups led by G Vainikko. The work is pursued all the way from mathematical theory via numerical methods to practical applications. A remarkable number of young scientists have got an opportunity to reach interesting results, and new directions for future work are outlined. In view of this total picture we would rate the research as a whole as excellent.

# Recommendations

Support to the group is strongly recommended. Support to the total research efforts led by G Vainikko is most strongly recommended.

Heino Türnpu Chair of Mathematical Analysis Tartu University

Linear Topological Properties of Spaces of Sequences or Operators

# Principal activities

Most of the members of this group of nine study the theory of summability, a generalization of the concept of convergence of a sequence or a series that is important in the theory of Fourier series. The exception is E Oja, who has brought geometric Banach space theory to Estonia.

The head of the chair, H Türnpu has for many years worked on various aspects of approximation theory and summability theory. T Leiger studies summability theory from the point of view of functional analysis and sequence spaces, taking as his starting point a classical theorem of Mazur and Orlicz, often working in collaboration with the German mathematician J Boos. L Loone, V Soomer, E Jürimäe, E Kolk, and E Reimers all work in summability theory. E Oja has successfully broken away from this tradition. The main directions of her research are the geometrical structure of dual spaces, and reproducibility of linear-topological properties.

Many diploma papers have been written under the supervision of members of the group.

#### Evaluation

The research ranges from fair to very good. The most interesting work in the group is done by E Oja. She has produced a fairly large number of papers and a small book in the period from 1986 onwards, and she seems to have a good network of international contacts. Many of her papers are published in international journals. In view of her research potential she should perhaps be encouraged to concentrate on larger projects rather than to continue her rapid output of short notes. In our opinion she is a talented young mathematician who has done some interesting work, and from whom one can expect more in the years to come. T Leiger is also a productive mathematician who publishes internationally. His collaboration with German mathematicians has led to a contract with the University in Hagen in Germany.

#### Recommendations

Summability theory is a field that has a strong position in Estonia, thanks to the influence of the late G Kangro. The problem is that it is no longer an internationally active field for research, and it survives only in a few isolated places. This does not mean that it is not possible to do good work in the field, especially on its functional analytic aspects, but for the sound development of analysis in Estonia it is necessary that young researchers are encouraged to move into more modern parts of analysis and functional analysis. We strongly recommend support for the efforts in this direction.

Jaan Lellep Chair of Theoretical Mechanics Tartu University

Optimal Design of Plastic Structures and Investigation of the Behaviour of Non-elastic Beams, Plates and Shells

# Principal activities

This group, built up by Ülo Lepik, and now led by Jaan Lellep, has been very productive for a long time. Its specialty is in nonlinear material behaviour, especially dynamic analysis of plastic structures, but in recent years emphasis has been put on methods for optimization of structures. Several conferences and summer schools have been organized. The group has a strong publication record, with articles in local and Soviet, but also in international journals, as well as several monographs issued in recent years.

M Heinloo reported works on the optimization of constructions where the material has a varying Young modulus, with multilayer constructions regarded as a special, practically important case. G Olenev has studied buckling loading of rigid plastic materials, shell bending, as well as more complicated yield conditions. E Sakkov deals with post critical behaviour of plates and shells, as well as special cases of optimization. T Lepikult studies models of the dynamic behaviour of rigid plastic structures. S Hannus reported work on rigid plastic cylindrical shells, and optimization for shells with piecewise constant thickness. K Hein has described the optimization of rigid plastic plates in order to get a minimal volume, and also reported some numerical computations for different configurations. K Soonets reported on how to determine the thickness of a rigid plastic cylindrical shell to get minimal deflection.

#### Evaluation

The overall rating of this group is very good, regarding especially the strong publication record and the thorough study of some pertinent and practically important problem areas.

#### Recommendations

We strongly recommend support for this group. It has a full agenda of future problems to study, and sees further developments both in theoretical and practical directions. It is in need of much better computing resources in order to fulfil this. A better recruitment of young research workers is needed if all possible developments are to be realized.

Ülo Lumiste Chair of Algebra and Geometry and Laboratory of Applied Mathematics Tartu University

Differential Geometry and Applications

# Principal activities

The group is centered around Ülo Lumiste, a geometer of international stature. His interests have ranged from the theory of principal bundles and connections on them to, more recently, the theory of semi-symmetric embeddings. M Rahula works on invariants of singularities of smooth mappings and related topics. V Abramov's research is in the part of differential geometry connected with modern mathematical physics (string theory). K Riives deals with the classification of Lie subgroups of the group of rigid motions of low-dimensional space and related homogeneous spaces, and A Parring works on symplectic submanifolds of symplectic space.

#### Evaluation

The group's work as a whole is to be rated as ranging from very good to excellent. However, the average age of the members of this group is fairly high, and unless something is done about recruiting new members, the group will face serious problems in the not too distant future.

#### Recommendations

We strongly recommend that this group be given the means to recruit students and young researchers.

Mati Kilp Chair of Algebra ared Geometry and Laboratory of Applied Mathematics Tartu University

Investigation of A 7 gebraic Structures and Their Representations

# Principal activities

The main area of interest of this group is the theory of monoids and similar algebraic structures. M Kilp works in the theory of monoids, investigating e.g. its homological properties. U Nummert investigates the structure of the monoid of endomorphisms of a graph. K Kaarli studies the theory of acts over monoids but has also more recently turned to the area of universal algebra. U Kaljulaid has a varied production in monoid theory, representation theory and rings with polynomial identities. He has also been interested in the history of Estonian mathematics. R Roomeldi has done some interesting work on implementing algorithms of algebra on computers. V Fleischer has been working on the theory of wreath products of monoids and also on a generalisation which involves categories instead of monoids.

# Evaluation

The group has demonstrated that it has a considerable competence in the field of algebra, but a substantial part of their work has been done in fields which are no longer of current interest. Their preoccupation with the theory of monoids is an example of the isolation which has affected some of Estonian mathematics, as it is no longer a very active area of research. On the other hand, some attempts has been made by members of the group to diversify and expand their interests. One such direction is the natural one of universal algebra, which at present has more roads open for further investigation, and which also seems to be experiencing some revival as a computational model. Another, at present very natural direction, is the one of algorithmic algebra, the working out of explicit and useful algorithms for computing with various algebraic structures, which has also been taken up by members of the group. Particularly U Kaljulaid is to be commended for his varied interests including the history of mathematics and the teaching of it. The work of the group ranges from fair to good.

# Recommendations

We strongly urge this group to try to further extend their research into areas of more current interest.

Rein Prank Laboratory of Applied Mathematics Tartu University

Teaching Programs for Computerized Exercises of Basic Courses in Mathematics

# Principal activities

This group has been working on developing computer programs to be used in the teaching of mathematics, specifically in logic and linear algebra. This is a very new field where very little is known. The program for teaching of logic has already been used successfully. It seemed to us to be very well thought through and executed, and avoiding many of the pitfalls which many programs for teaching of mathematics have fallen into. The program for linear algebra looked promising, but it seems too early to tell its success, since it has not yet been used in teaching. (This test seems to be particularly necessary for computer programs used for the teaching of mathematics as opposed to more traditional media).

# **Evaluation**

The work of the group stands very well in international comparison and should be judged very good.

# Recommendations

We strongly recommend that this group be given further support.

Mati Abel Laboratory of Applied Mathematics Tartu University

Topological Algebras

# Principal activities

This group with two members has mainly been working on extending known results for special topological algebras such as Banach algebras to more general ones. It is a classical theorem that the only Banach division algebra is the complex numbers, and M Abel investigates extensions of this result to algebras which are as general as possible. He also has some results in the same vein on commutative algebras and stability under projective limits. A Kokk works on generalizing the notions of spectrum and joint spectrum.

#### Evaluation

The current trend in the area of topological algebras is to study more in detail algebras with richer structure, such as C\*-algebras, which also have important applications in physics. The group would do well to try to follow this trend. The work ranges from fair to good.

#### Recommendations

We recommend that the group be given support in the same way as teachers associated with chairs.

Ene-Margit Tiit, Kuldev Aaremaa, and Liina-Mai Tooding Department of Mathematical Statistics and Laboratory of Applied Mathematics Tartu University

Investigation, Elaboration and Application of Computational Statistical Models

# Principal activities

The activities of this project have mainly been connected with the development, testing, and using of statistical software. Original statistical packages have been created, but also Western commercial packages have been modified to be used on the computers available.

The group has every year been involved in analysing great numbers of empirical data sets from various sciences such as medicine, biology, sociology, psychology, education, etc. Besides serving scientists working at Tartu University, the group also serves scientists from research institutes of the Estonian Academy of Sciences and other institutes of the former Soviet Union. The research staff - formerly belonging to the Section of Statistics and Data Analysis of the Computer Centre of Tartu University - has consisted of about ten persons: mathematicians, statisticians, programmers and a manager.

Inspired and connected with the activities above, more "theoretical" research has also been done, especially on multivariate statistical analysis. Methods for testing the correctness of computational procedures connected with such methods have been investigated. Results on clustering methods have been obtained. Also the activities of the project "Estimation in Multivariate Statistical Models" (see next page) are connected with this.

#### Evaluation

It is important for scientists in many different areas to get competent advice and help in analysing empirical data. Both statistical competence and knowledge of the use of computers are needed. The group has extensive experience and know-how in these matters, and has apparently been successful in developing good contacts with researchers in many areas. These contacts have also created interesting research problems within statistics. The group has an important role in the education of students at Tartu University, as some of these do project work connected with the consultation activities of the group. It is a scientifically good project.

# Recommendations

Support for this group is strongly recommended. It is very important for Tartu University, and for Estonia, to have qualified specialists in statistical data analysis. For the group to maintain high standards it is necessary that its members also get time for their own research.

Tönu Kollo, Tönu Möls, and Kalev Pärna
Department of Mathematical Statistics and
Laboratory of Applied Mathematics
Tartu University
Group of Biometrics
Institute of Zoology and Botany
Estonian Academy of Sciences
Tartu

Estimation in Multivariate Statistical Models

# Principal activities

Research in multivariate analysis has been performed in the following directions: deriving asymptotic distributions, multivariate Edgeworth expansions, constructing multivariate distributions with given marginals, moments, etc, approximating a distribution with a discrete one, development of multivariate analysis in environmental studies.

Results have been obtained in a unified way using matrix derivative techniques. Asymptotic normality of statistics have been established for a wide class of population distributions; this is important for the application of multivariate methods to empirical data. Investigation on classification problems has lead to research on approximation of distributions in metric spaces. In a project concerning monitoring and environmental studies of lakes, the group has developed approximations of multivariate distributions. Computer simulation has been used for checking theoretical results.

The research has been carried out in close cooperation with many other research centers both in the West and in the former Soviet Union.

# Evaluation

The activities of this group are connected with, and have partly grown out of the activities of the project "Investigation, Elaboration and Application of Computational Statistical Models" (see previous page); they are different sides of the same coin. Getting reliable approximations in multivariate analysis is important for the applicability of such methods to empirical data. The matrix derivative technique used gives a nice unification in the mathematical analysis of the problems. The close connection with important environmental problems is interesting. It is a scientifically good project.

#### Recommendations

This project is very much connected with the project "Investigation, Elaboration and Application of Computational Statistical Models" (see previous page); persons are working in both projects. The conclusions are the same for both projects. Support is strongly recommended.

Jaak Sikk Department of Mathematics Estonian Agricultural Academy Tartu

Rate-spaces in the Theory of Summability

# Principal activities

J Sikk is the only member of this group, but he works in the same field, the theory of summability, as many other Tartu mathematicians. He has no particular resources for research, so his research is essentially done in his spare time, and our comments have to be seen in this light. We feel however, that the direction of his research, which consists in extending summability methods to even more general sequence spaces, is not very fruitful and will not give any deep results. The results he has obtained so far are published in a short note in the Tartu journal.

#### Evaluation

We consider this research to be fair.

#### Recommendations

We have the opinion that it is important that a department like this one, where mathematics is mainly a service subject, has teachers who are active in research, and that such teachers are given at least some time for research. We think it would be valuable if J. Sikk could redirect his considerable enthusiasm for his subject in a more fruitful direction.

I Petersen
Department of Mathematics
Institute of Cybernetics
Estonian Academy of Sciences
Tallinn

Applied Mathematics

# Principal activities

This research group, consisting of 8 senior research fellows, reported on work in three areas: numerical methods (and harmonic analysis), stochastic programming, and mathematical modelling. Some of the fellows also give lectures on mathematical analysis and operations research at the Tallinn Technical University. One of the roles of the department has been consultations on mathematics in general and in operations research and statistics from other departments, research institutes and governmental offices. Several symposia have been arranged. The department has earned money through contracts on testing software for numerical methods, developing systems of optimizing mixtures and by developing and selling statistical software packages.

# Numerical methods

O Vaarmann studies nonlinear least squares problems and has used Gauss-Newton, and Levenberg-Marquardt stabilisation, as well as homotopy continuation methods. Conditions for convergence as well as estimates of speed of convergence are derived, and in some cases results of numerical tests are reported.

I Keis reported on methods for aggregation and decomposition of large control systems, especially cases where fast and slow modes are treated separately. Stabilization of large mechanical systems by modifying a part of the Hamiltonian can be described in this way.

J Lippus has extended a classical Fourier multiplier theorem of A Zygmund to more general function classes.

# Stochastic programming

E Tamm has worked on one-stage stochastic programming problems where a mathematical expectation or a probability is minimized under expectational or probabilistic constraints. The results are published in a few short notes.

In the seventies the American mathematicians J T Rockafellar and R Wets studied convex stochastic programming problems with recourse. R Lepp has in a series of papers investigated one possibility of getting approximate solutions of such problems. Lepp has 18 published papers 1986-1990, some of them in international journals of high reputation. Lepp is an active scientist working in an interesting area.

# Mathematical modelling

I Petersen and V Olman have in recent years investigated problems in mathematical modelling of e.g. multidecisions. Petersen has in a number of papers given general formulations, conditions for existence and solution of such problems. Olman has solved particular problems.

The multidecision approach has been used by Petersen to model the multi-party

democracy. It was shown that individual and collective freedoms of choice are sufficient properties to guarantee the democratic development of a society to a local optimum. We are not sure that mathematics can be used to "prove" such things about society.

# Evaluation

The research ranges from fair to very good.

# Recommendations

We think this research group should be integrated with the Department of Mathematics of the Tallinn Technical University, and that the teaching and research duties should be distributed more according to the proven abilities of the researchers.

Kalju Kenk, Jüri Kirs, and Heino Relvik Chair of Foundations of Mechanics Tallinn Technical University

Equations of Movement and State of Mechanic Systems, and Optimal Design of Plastic Elements of Constructions

# Principal activities

The scientific work of this group is closely related to the corresponding group in Tartu (see page 10), but its main responsibility is teaching. J Kirs reported on work concerning the optimal design of plastic plates with special consideration of the hardening of material. K Kenk and A Salupere also consider plastic materials, while G Arjassov studies oscillations of mechanical systems, especially contact constraints. H Relvik has developed and investigated an alternative way of describing the motion of a mechanical system, the Equations of Possible Power, and A Haitin has studied some special cases in this context.

#### **Evaluation**

The rating of this work is good, especially if account is taken of the fact that it has been done parallel to a full time teaching program.

#### Recommendations

It is important that those who teach mechanics at the Tallinn Technical University are also active in research, and are given time for this. Support is recommended.

L Ainola, H Päeva, A Kivinukk, L Sarv, and A Löhmus 1epartment of Mathematics Tallinn Technical University

Applied Methods of Mathematical Analysis, Algebra and Geometry

# Principal Activities

The mathematical research of the department can be roughly divided into three groups: algebra and geometry, summability methods, numerical methods.

Algebra and geometry

H Päeva works on the subject of monoids, closely allied to the group of Tartu University. E Paal works in the theory of topological Moufang loops, T Virovere and M Väljas work on problems in the differential geometry of submanifolds in the tradition of the interests of Ü Lumiste. V Orlov studies the application of group theory to the classical theory of hydrodynamics.

Summability methods

A Kivinukk and others work in the theory of approximation and summability in the Estonian tradition. Some practical numerical approximation methods have also been developed.

Numerical methods

This group is the largest one. Among others G Dumkina reported on a turbulent flow problem, E Pais on a plate eigenvalue problem, A Pirozhenko on a large system of ordinary differential equations coming from modelling a semiconductor device. L Sarv reported a series of works on water network simulation, water cleaning and liquid motion in pipes, as well as new algorithms for ill-posed linear problems.

#### **Evaluation**

We can only give a rather general evaluation of the very diverse research of this department. We are impressed with the fact that so many of its members are active in research in spite of a very large teaching load. The readiness of several members of the group to take up work on applied problems is to be especially commended. Naturally the work varies in quality, but our general impression is that it is good and competent.

#### Recommendations

Since it seems that the group has essentially no resources for research it is easy to make recommendations. We think it is very important for the future of the Tallinn Technical University to have a strong Mathematics Department, which in addition to being responsible for teaching is able to build up fundamental knowledge and to cooperate with users of mathematics. We recommend strongly that the Department of Mathematics of the Institute of Cybernetics be integrated with this department, and that a mechanism be found for distributing the economic resources which are available for research to those who can make the best use of it.

# Peeter Normak Tallinn Teacher Training Institute

Acts over Monoids

# Principal activities

The group consists only of Peter Normak. The Tallinn Teacher Training Institute is an educational institution for the education of future school teachers. The teaching load is fairly high (about 14 hours a week). Mathematically Normak belongs to the group centered at Tartu University working in the area of the theory of monoids and actions of monoids on sets. He has fairly well established international contacts and has done competent work in his chosen area.

#### Evaluation

The quality of this work is to be considered as good.

# Recommendations

In view of the role played by the Tallinn Teacher Training Institute in the education of future mathematics teachers it is important that it has teachers who are active in research. We regret that only one of its teachers took part in the evaluation. We recommend that Normak be given support for research.

#### **APPENDIX**

# Background of evaluators

# Torsten Ekedahl

Professor in Mathematics at Stockholm University since 1988. His current research interests involve the area of the interplay between arithmetic and geometry and particularly the arithmetic interpretation of the cohomology of algebraic varieties.

# Lars Holst

Professor of Mathematical Statistics, Royal Institute of Technology, Stockholm. His current research interests are connected with Poisson and related approximations in probability theory, especially for combinatorial and geometric problems. He is also involved in different applications of probability and statistics in science and technology.

# Lars Inge Hedberg

Professor and Head of Department of Mathematics, Linköping University. His main research interests are the theory of function spaces, potential theory, and harmonic analysis.

#### Axel Ruhe

Professor of Numerical Analysis at Chalmers University of Technology and the University of Göteborg, dean of the School of Mathematical and Computing Sciences there. Research in numerical linear algebra, especially numerical algorithms for eigenvalue problems and nonlinear parameter estimation.